Simulation and Noise

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Problem Setup

Question. How well can we simulate open quantum system dynamics by unitary gates on a quantum computer?

Evolution of quantum systems are described in terms of quantum channels, or completely positive and trace-preserving maps. One of the motivating idea for the construction of quantum computer is to simulate particular dynamics of quantum evolution in the lab.
In this project, we consider a quantum dynamical semigroup and the cost of simulating the channels given by a specified resource set.
Let $(T_t)_{t\geq 0} = (e^{-tL})_{t\geq 0}$ be a one-parameter semigroup acting on n qubits, where

$$L = \sum_{j=1}^{n} L_{a_j}$$

is a Lindbladian operator consisting of local components given by

$$L_a(\rho) = a^*a\rho + \rho a^*a - 2a\rho a^*$$

and

$$a_j = I \otimes \cdots \otimes I \otimes a \otimes I \otimes \cdots \otimes I$$

have a on the j-th component and identity everywhere else.
We will consider two concrete examples:

- **LCU Model**: Let $L = \sum_j L_{X_j} + L_{Z_j}$. The channel e^{-tL} is the product of X and Z rotations. The resource set is given by $\mathcal{U} = \left\{e^{itX_j} : 0 \leq t \leq 2\pi\right\} \cup \left\{e^{itZ_j} : 0 \leq t \leq 2\pi\right\}$
- **Photon Emission Model**: Let $L = \sum L_{a_j}$ where $a = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

Simulation Cost and Resource-Dependent Complexity

We will identify the useful resource set and the corresponding unitary set $\mathcal{U}$ in addition to the help of classical averaging to approximate the channel by

$$\Phi_\mu(|\psi\rangle\langle\psi|) = \mathbb{E}_\mu U_m \ldots U_1 |\psi\rangle\langle\psi| U_1^* \ldots U_m^* \quad \text{where} \quad U_1, \ldots, U_m \in \mathcal{U}$$

Question: For given $\delta > 0$ and a fix t, what is the minimal depth m such that

$$||\Phi_\mu - e^{-tL}||_\diamond < \delta.$$

We use standard diamond norm as measure of approximation. Then we minimize over the depth m:

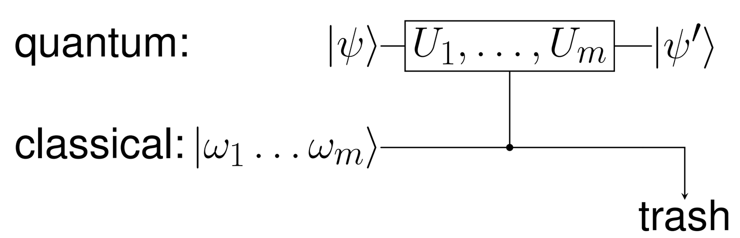
$$l_\delta^{\mathcal{U}}(e^{-tL}) = \inf\left\{m : ||\Phi_\mu - e^{-tL}||_\diamond < \delta \text{ for some measure } \mu\right\}.$$

For small t we are close to identity. So we require better approximations in a uniform way by considering the maximal simulation length for the correlated error rate $\delta = \alpha t^\beta$ as

$$M_{\alpha,\beta} = \sup_{t>0} l_{\alpha t^\beta}^{\mathcal{U}}(T_t).$$

Classically-Assisted Depth m

We want to find the minimal number of unitaries required to simulate the evolution with the help of some classical resources (averaging) up to the fixed error. The following is an demonstration of the idea in terms of circuits.



Via this framework, we are able to produce both lower and upper estimates for the maximal simulation length.

LCU Model

For the LCU model we first find a good gaussian average for the local Lindbladians. We combine this with a well-known composition trick due to Suzuki. This allows us to show for the combined X, Z Lindbladian that

$$M_{64n,2} \leq 2n \quad \text{and} \quad M_{1,2} \leq 128n^2$$

and

$$M_{512n,3} \leq 3n \quad \text{and} \quad M_{1,3} \leq 1536n^2$$

Suzuki proved that for error estimate of degree higher than 3, one will always overshoot and thus the semigroup framework will not have a bound for the length.
Lower bounds are much harder to prove. We can steel a recent complexity concept for channels. In the LCU-X, Z model this new maximal cost is of order n, the number of qubits. For the LCU model, it is possible to explicitly bound. Hence,
When $\beta = 2$,

$$\frac{n}{128\pi} \leq M_{1,2} \leq 128n^2$$

When $\beta = 3$,

$$\frac{\sqrt{n}}{1024\pi} \leq M_{512n,3} \leq 3n$$

Photon Emission Model

For the photon emission model, we need to consider dilation of the system

$$T_t(\rho) = \mathrm{tr}_E(U(|0\rangle\langle 0| \otimes \rho)U^*)$$

and calculate the unitary U explicitly. In the particular case,

$$U = e^{itH} \quad \text{where} \quad H = \frac{Y \otimes X - X \otimes Y}{2}$$

Hence, it is possible to use 2 gates to achieve zero-error simulation and obtain

$$M_{\alpha,\beta} \leq 2n.$$

The same lower estimate as of the LCU case holds with the resource set

$$S_{EA} = \{I, X, Y, Z\} \otimes \{I, X, Y, Z\} \quad \text{and} \quad \mathcal{U} = \left\{e^{its}, s \in S_{EA}\right\}$$

Comments

1. Before the start of the IML project, $M_{\alpha,\beta}$ in the photon emission model was neither well-defined nor known to be finite.
2. It is possible to obtain a similar upper bound for the photon emission model if we take the resource set as
$$\mathcal{U}' = \left\{C - e^{itZ}, H, SWAP, CNOT\right\}$$
up to a constant.
3. Further research will give information about what the "bad" time t is. That means, what are the times that are hard to approximate with small depth.
4. With our particular choice of unitaries, the maximal depth of the random circuits is a scaling problem.
5. For upper estimates, we can replace the continuous expectation by a finite average of depth m gates.
6. In our LCU model the approximating unitaries are X and Z rotations, which can be programmed in IBM machine and other platforms.

Open Problems (In Progress)

1. Find good unitaries for the case where $L = \sum_j L_{a_j}$ and the a_j are non-Hermitian. The need for extra environment makes the canoncial choice of good unitaries and the tools from convex complexity harder to adapt for true, noisy open system models.
2. Extending lower and upper bound for even larger classes of noise models beyond commuting ones.

References

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